# Package: tinyVAST (via r-universe)

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Type Package

Title Multivariate Spatio-Temporal Models using Structural Equations

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Description Fits a wide variety of multivariate spatio-temporal models with simultaneous and lagged interactions among variables (including vector autoregressive spatio-temporal ('VAST') dynamics) for areal, continuous, or network spatial domains. It includes time-variable, space-variable, and space-time-variable interactions using dynamic structural equation models ('DSEM') as expressive interface, and the 'mgcv' package to specify splines via the formula interface. See Thorson et al. (2024) <doi:10.48550/arXiv.2401.10193> for more details.

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**Depends** R (>= 4.1.0)

**Imports** corpcor, fmesher, igraph, Matrix (>= 1.3.0), methods, mgcv, sem, sf, sfnetworks, TMB (>= 1.9.17), units, checkmate, abind, sdmTMB

**Suggests** dsem (>= 1.1.0), ggplot2, knitr, lattice, mvtnorm, pdp, rmarkdown, rnaturalearth, rnaturalearthdata, testthat, tweedie, viridisLite, visreg, plyr, DHARMa, glmmTMB, tibble

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URL https://vast-lib.github.io/tinyVAST/

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# BugReports https://github.com/vast-lib/tinyVAST/issues

**Config/pak/sysreqs** cmake libgdal-dev gdal-bin libgeos-dev libglpk-dev make libicu-dev libxml2-dev libssl-dev libproj-dev libsqlite3-dev libudunits2-dev

Repository https://vast-lib.r-universe.dev

RemoteUrl https://github.com/vast-lib/tinyvast

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## Description

Given user-provided newdata, expand the object tmb\_data to include predictions corresponding to those new observations

## Usage

```
add_predictions(object, newdata, remove_origdata = FALSE)
```

## Arguments

object	Output from tinyVAST().
newdata	New data-frame of independent variables used to predict the response.
remove_origdat	a
	Whether to remove original-data to allow faster evaluation. remove_origdata=TRUE eliminates information about the distribution for random effects, and cannot be combined with epsilon bias-correction. WARNING: feature is experimental and subject to change.

#### Value

the object fit\$tmb\_inputs\$tmb\_data representing data used during fitting, but with updated values for slots associated with predictions, where this updated object can be recompiled by TMB to provide predictions

bering\_sea

Survey domain for the eastern and northern Bering Sea surveys

## Description

Shapefile defining the spatial domain for the eastern and northern Bering Sea bottom trawl surveys.

#### Usage

data(bering\_sea)

bering\_sea\_pollock\_ages

Survey catch-rates at age for Alaska pollock in the Eastern and Northern Bering Sea

#### Description

Data used to demonstrate and test model-based age expansion, using density= dependence corrected survey catch rates after first=stage expansion from the bottom trawl survey for ages 1-15, conducted by by the Alaska Fisheries Science Center, including annual surveys in the eastern Bering Sea 1982-2019 and 2021-2023, as well as the northern Bering Sea in 1982/85/88/91 and 2010/17/18/19/21/22/23.

#### Usage

```
data(bering_sea_pollock_ages)
```

bering\_sea\_pollock\_vast

Estimated proportion-at-age for Alaska pollock using VAST

## Description

Estimated proportion-at-age for Alaska pollock using the package VAST, for comparison with output using tinyVAST.

## Usage

```
data(bering_sea_pollock_vast)
```

cAIC

Calculate conditional AIC

## Description

Calculates the conditional Akaike Information criterion (cAIC).

#### Usage

```
cAIC(object)
```

#### Arguments

object Output from tinyVAST().

### cAIC

## Details

cAIC is designed to optimize the expected out-of-sample predictive performance for new data that share the same random effects as the in-sample (fitted) data, e.g., spatial interpolation. In this sense, it should be a fast approximation to optimizing the model structure based on k-fold cross-validation.

By contrast, AIC() calculates the marginal Akaike Information Criterion, which is designed to optimize expected predictive performance for new data that have new random effects, e.g., extrapolation, or inference about generative parameters.

Both cAIC and EDF are calculated using Eq. 6 of Zheng, Cadigan, and Thorson (2024).

For models that include profiled fixed effects, these profiles are turned off.

#### Value

cAIC value

## References

Zheng, N., Cadigan, N., & Thorson, J. T. (2024). A note on numerical evaluation of conditional Akaike information for nonlinear mixed-effects models (arXiv:2411.14185). arXiv. doi:10.48550/arXiv.2411.14185

#### Examples

```
data( red_snapper )
red_snapper = droplevels(subset(red_snapper, Data_type=="Biomass_KG"))
# Define mesh
mesh = fmesher::fm_mesh_2d( red_snapper[,c('Lon', 'Lat')],
                           cutoff = 1)
# define formula with a catchability covariate for gear
formula = Response_variable ~ factor(Year) + offset(log(AreaSwept_km2))
# make variable column
red_snapper$var = "logdens"
# fit using tinyVAST
fit = tinyVAST( data = red_snapper,
                formula = formula,
                sem = "logdens <-> logdens, sd_space",
                space_columns = c("Lon", 'Lat'),
                spatial_graph = mesh,
                family = tweedie(link="log"),
                variable_column = "var",
                control = tinyVASTcontrol( getsd = FALSE,
                                           profile = "alpha_j" ) )
cAIC(fit) # conditional AIC
AIC(fit) # marginal AIC
```

#### Description

classify\_variables is copied from sem:::classifyVariables

#### Usage

```
classify_variables(model)
```

#### Arguments

model syntax for structural equation model

## Details

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#### Value

Tagged-list defining exogenous and endogenous variables

condition\_and\_density Condition and density example

# Description

Data used to demonstrate and test a bivariate model for morphometric condition (i.e., residuals in a weight-at-length relationship) and density for fishes, using the same example as was provided as a wiki example for VAST. Data are from doi:10.3354/meps13213

## Usage

data(condition\_and\_density)

#### Description

deviance\_explained fits a null model, calculates the deviance relative to a saturated model for both the original and the null model, and uses these to calculate the proportion of deviance explained.

This implementation conditions upon the maximum likelihood estimate of fixed effects and the empirical Bayes ("plug-in") prediction of random effects. It can be described as "conditional deviance explained". A state-space model that estimates measurement error variance approaching zero (i.e., collapses to a process-error-only model) will have a conditional deviance explained that approaches 1.0

#### Usage

```
deviance_explained(x, null_formula, null_delta_formula = ~1)
```

#### Arguments

х	<pre>output from \code{tinyVAST()}</pre>		
null_formula	formula for the null model. If missing, it uses null_formula = response ~ 1. For multivariate models, it might make sense to use null_formula = response ~ category		
null_delta_formula			
	<pre>formula for the null model for the delta component. If missing, it uses null_formula = response ~ 1. For multivariate models, it might make sense to use null_delta_formula = response ~ category</pre>		

## Value

the proportion of conditional deviance explained.

Families Ada

Additional families

## Description

Additional families compatible with tinyVAST().

#### Usage

```
delta_lognormal(link1, link2 = "log", type = c("standard", "poisson-link"))
delta_gamma(link1, link2 = "log", type = c("standard", "poisson-link"))
```

#### Arguments

link1	Link for first part of delta/hurdle model.
link2	Link for second part of delta/hurdle model.
type	Delta/hurdle family type. "standard" for a classic hurdle model. "poisson-link" for a Poisson-link delta model (Thorson 2018).
link	Link.

#### Value

A list with elements common to standard R family objects including family, link, linkfun, and linkinv. Delta/hurdle model families also have elements delta (logical) and type (standard vs. Poisson-link).

## References

Poisson-link delta families:

Thorson, J.T. 2018. Three problems with the conventional delta-model for biomass sampling data, and a computationally efficient alternative. Canadian Journal of Fisheries and Aquatic Sciences, 75(9), 1369-1382. doi:10.1139/cjfas20170266

#### Poisson-link delta families:

Thorson, J.T. 2018. Three problems with the conventional delta-model for biomass sampling data, and a computationally efficient alternative. Canadian Journal of Fisheries and Aquatic Sciences, 75(9), 1369-1382. doi:10.1139/cjfas20170266

#### Examples

delta\_lognormal()
delta\_gamma()

integrate\_output Integration for target variable

## Description

Calculates an estimator for a derived quantity by summing across multiple predictions. This can be used to approximate an integral when estimating area-expanded abundance, abundance-weighting a covariate to calculate distribution shifts, and/or weighting one model variable by another.

#### Usage

```
integrate_output(
   object,
   newdata,
   area,
   type = rep(1, nrow(newdata)),
```

# integrate\_output

```
weighting_index,
covariate,
getsd = TRUE,
bias.correct = TRUE,
apply.epsilon = FALSE,
intern = FALSE
)
```

# Arguments

object	Output from tinyVAST().		
newdata	New data-frame of independent variables used to predict the response, where a total value is calculated by combining across these individual predictions. If these locations are randomly drawn from a specified spatial domain, then integrate_output applies midpoint integration to approximate the total over that area. If locations are drawn sysmatically from a domain, then integrate_output is applying a midpoint approximation to the integral.		
area	vector of values used for area-weighted expansion of estimated density surface for each row of newdata with length of nrow(newdata).		
type	Integer-vector indicating what type of expansion to apply to each row of newdata, with length of nrow(newdata).		
	type=1 Area-weighting: weight predictor by argument area		
	<pre>type=2 Abundance-weighted covariate: weight covariate by proportion of total in each row of newdata</pre>		
	type=3 Abundance-weighted variable: weight predictor by proportion of total in a prior row of newdata. This option is used to weight a prediction for one category based on predicted proportional density of another category, e.g., to calculate abundance-weighted condition in a bivariate model.		
	type=4 Abundance-expanded variable: weight predictor by density in a prior row of newdata. This option is used to weight a prediction for one category based on predicted density of another category, e.g., to calculate abundance- expanded consumption in a bivariate model.		
	type=0 Exclude from weighting: give weight of zero for a given row of newdata. Including a row of newdata with type=0 is useful, e.g., when calculating abundance at that location, where the eventual index uses abundance as weighting term but without otherwise using the predicted density in calcu- lating a total value.		
weighting_index			
	integer-vector used to indicate a previous row that is used to calculate a weighted average that is then applied to the given row of newdata. Only used for when type=3.		
covariate	numeric-vector used to provide a covariate that is used in expansion, e.g., to pro- vide positional coordinates when calculating the abundance-weighted centroid with respect to that coordinate. Only used for when type=2.		
getsd	logical indicating whether to get the standard error, where getsd=FALSE is faster during initial exploration		

bias.correct	logical indicating if bias correction should be applied using standard methods in $TMB::sdreport()$
apply.epsilon	Apply epsilon bias correction using a manual calculation rather than using the conventional method in TMB::sdreport? See details for more information.
intern	Do Laplace approximation on C++ side? Passed to TMB::MakeADFun().

#### **Details**

Analysts will often want to calculate some value by combining the predicted response at multiple locations, and potentially from multiple variables in a multivariate analysis. This arises in a univariate model, e.g., when calculating the integral under a predicted density function, which is approximated using a midpoint or Monte Carlo approximation by calculating the linear predictors at each location newdata, applying the inverse-link-trainsformation, and calling this predicted response mu\_g. Total abundance is then be approximated by multiplying mu\_g by the area associated with each midpoint or Monte Carlo approximation point (supplied by argument area), and summing across these area-expanded values.

In more complicated cases, an analyst can then use covariate to calculate the weighted average of a covariate for each midpoint location. For example, if the covariate is positional coordinates or depth/elevation, then type=2 measures shifts in the average habitat utilization with respect to that covariate. Alternatively, an analyst fitting a multivariate model might weight one variable based on another using weighting\_index, e.g., to calculate abundance-weighted average condition, or predator-expanded stomach contents.

In practice, spatial integration in a multivariate model requires two passes through the rows of newdata when calculating a total value. In the following, we write equations using C++ indexing conventions such that indexing starts with 0, to match the way that integrate\_output expects indices to be supplied. Given inverse-link-transformed predictor  $\mu_q$ , function argument type as  $type_q$  function argument area as  $a_q$ , function argument covariate as  $x_q$ , function argument weighting\_index as  $eqn{h_g}$  function argument weighting\_index as  $eqn{h_g}$  the first pass calculates:

$$\nu_g = \mu_g a_g$$

where the total value from this first pass is calculated as:

$$\nu^* = \sum_{q=0}^{G-1} \nu_g$$

The second pass then applies a further weighting, which depends upon  $type_g$ , and potentially upon  $x_g$  and  $h_g$ .

If  $type_q = 0$  then  $\phi_q = 0$ 

If 
$$type_a = 1$$
 then  $\phi_a = \nu_a$ 

If  $type_g = 1$  then  $\phi_g = \nu_g$ If  $type_g = 2$  then  $\phi_g = x_g \frac{\nu_g}{\nu^*}$ 

If 
$$type_a = 3$$
 then  $\phi_a = \frac{\nu_{h_g}}{\pi}\mu$ 

If  $type_g = 3$  then  $\phi_g = \frac{\nu_{h_g}}{\nu^*} \mu_g$ If  $type_g = 4$  then  $\phi_g = \nu_{h_g} \mu_g$ 

Finally, the total value from this second pass is calculated as:

$$\phi^* = \sum_{g=0}^{G-1} \phi_g$$

and  $\phi^*$  is outputted by integrate\_output, along with a standard error and potentially using the epsilon bias-correction estimator to correct for skewness and retransformation bias.

Standard bias-correction using bias.correct=TRUE can be slow, and in some cases it might be faster to do apply.epsilon=TRUE and intern=TRUE. However, that option is somewhat experimental, and a user might want to confirm that the two options give identical results. Similarly, using bias.correct=TRUE will still calculate the standard-error, whereas using apply.epsilon=TRUE and intern=TRUE will not.

## Value

A vector containing the plug-in estimate, standard error, the epsilon bias-corrected estimate if available, and the standard error for the bias-corrected estimator. Depending upon settings, one or more of these will be NA values, and the function can be repeatedly called to get multiple estimators and/or statistics.

logLik.tinyVAST Extract the (marginal) log-likelihood of a tinyVAST model

#### Description

Extract the (marginal) log-likelihood of a tinyVAST model

## Usage

```
## S3 method for class 'tinyVAST'
logLik(object, ...)
```

#### Arguments

object	output from tinyVAST
	not used

#### Value

object of class logLik with attributes

val	log-likelihood
df	number of parameters

make\_dsem\_ram

# Description

make\_dsem\_ram converts SEM arrow notation to ram describing SEM parameters

#### Usage

```
make_dsem_ram(
 dsem,
  times,
  variables,
  covs = NULL,
 quiet = FALSE,
  remove_na = TRUE
)
```

## Arguments

dsem	dynamic structural equation model structure, passed to either specifyModel or specifyEquations and then parsed to control the set of path coefficients and variance-covariance parameters		
times	A character vector listing the set of times in order		
variables	A character vector listing the set of variables		
covs	optional: a character vector of one or more elements, with each element giv- ing a string of variable names, separated by commas. Variances and covariances among all variables in each such string are added to the model. For confirmatory factor analysis models specified via cfa, covs defaults to all of the factors in the model, thus specifying all variances and covariances among these factors. <i>Warn- ing</i> : covs="x1, x2" and covs=c("x1", "x2") are <i>not</i> equivalent: covs="x1, x2" specifies the variance of x1, the variance of x2, <i>and</i> their covariance, while covs=c("x1", "x2") specifies the variance of x1 and the variance of x2 <i>but not</i> their covariance.		
quiet	Boolean indicating whether to print messages to terminal		
remove_na	Boolean indicating whether to remove NA values from RAM (default) or not. remove_NA=FALSE might be useful for exploration and diagnostics for advanced users		

## Details

## RAM specification using arrow-and-lag notation

Each line of the RAM specification for make\_dsem\_ram consists of four (unquoted) entries, separated by commas:

- Arrow specification: This is a simple formula, of the form A -> B or, equivalently, B <- A for a regression coefficient (i.e., a single-headed or directional arrow); A <-> A for a variance or A <-> B for a covariance (i.e., a double-headed or bidirectional arrow). Here, A and B are variable names in the model. If a name does not correspond to an observed variable, then it is assumed to be a latent variable. Spaces can appear freely in an arrow specification, and there can be any number of hyphens in the arrows, including zero: Thus, e.g., A->B, A --> B, and A>B are all legitimate and equivalent.
- **2. Lag (using positive values):** An integer specifying whether the linkage is simultaneous (lag=0) or lagged (e.g., X -> Y, 1, XtoY indicates that X in time T affects Y in time T+1), where only one-headed arrows can be lagged. Using positive values to indicate lags then matches the notational convention used in package **dynlm**.
- **3. Parameter name:** The name of the regression coefficient, variance, or covariance specified by the arrow. Assigning the same name to two or more arrows results in an equality constraint. Specifying the parameter name as NA produces a fixed parameter.
- **4. Value:** start value for a free parameter or value of a fixed parameter. If given as NA (or simply omitted), the model is provide a default starting value.

Lines may end in a comment following #. The function extends code copied from package sem under licence GPL (>= 2) with permission from John Fox.

#### Simultaneous autoregressive process for simultaneous and lagged effects

This text then specifies linkages in a multivariate time-series model for variables **X** with dimensions  $T \times C$  for T times and C variables. make\_dsem\_ram then parses this text to build a path matrix **P** with dimensions  $TC \times TC$ , where  $\rho_{k_2,k_1}$  represents the impact of  $x_{t_1,c_1}$  on  $x_{t_2,c_2}$ , where  $k_1 = Tc_1 + t_1$  and  $k_2 = Tc_2 + t_2$ . This path matrix defines a simultaneous equation

$$\operatorname{vec}(\mathbf{X}) = \operatorname{Pvec}(\mathbf{X}) + \operatorname{vec}(\mathbf{\Delta})$$

where  $\Delta$  is a matrix of exogenous errors with covariance  $\mathbf{V} = \Gamma \Gamma^t$ , where  $\Gamma$  is the Cholesky of exogenous covariance. This simultaneous autoregressive (SAR) process then results in  $\mathbf{X}$  having covariance:

$$\operatorname{Cov}(\mathbf{X}) = (\mathbf{I} - \mathbf{P})^{-1} \mathbf{\Gamma} \mathbf{\Gamma}^{t} ((\mathbf{I} - \mathbf{P})^{-1})^{t}$$

Usefully, it is also easy to compute the inverse-covariance (precision) matrix  $\mathbf{Q} = \mathbf{V}^{-1}$ :

$$\mathbf{Q} = (\mathbf{\Gamma}^{-1}(\mathbf{I} - \mathbf{P}))^t \mathbf{\Gamma}^{-1}(\mathbf{I} - \mathbf{P})$$

#### Example: univariate and first-order autoregressive model

This simultaneous autoregressive (SAR) process across variables and times allows the user to specify both simultaneous effects (effects among variables within year T) and lagged effects (effects among variables among years T). As one example, consider a univariate and first-order autoregressive process where T = 4. with independent errors. This is specified by passing dsem = X -> X, 1, rho; X <-> X, 0, sigma to make\_dsem\_ram. This is then parsed to a RAM:

heads	to	from	paarameter	start
1	2	1	1	NA

1	3	2	1	NA
1	4	3	1	NA
2	1	1	2	NA
2	2	2	2	NA
2	3	3	2	NA
2	4	4	2	NA

Rows of this RAM where heads=1 are then interpreted to construct the path matrix P:

```
\deqn{ \mathbf P = \begin{bmatrix}
    0 & 0 & 0 & 0 \
    \rho & 0 & 0 & 0 \
    0 & \rho & 0 & 0 \
    0 & \rho & 0 & 0 \
    0 & 0 & \rho & 0 \
    \end{bmatrix} }
```

While rows where heads=2 are interpreted to construct the Cholesky of exogenous covariance  $\Gamma$ :

```
\deqn{ \mathbf \Gamma = \begin{bmatrix}
   \sigma & 0 & 0 & 0 \
   0 & \sigma & 0 & 0 \
   0 & 0 & \sigma & 0 \
   0 & 0 & \sigma & 0 \
   0 & 0 & 0 & \sigma \
   \end{bmatrix} }
```

with two estimated parameters  $\beta = (\rho, \sigma)$ . This then results in covariance:

```
\deqn{ \mathrm{Cov}(\mathbf X) = \sigma^2 \begin{bmatrix}
    1 & \rho^1 & \rho^2 & \rho^3 \
    \rho^1 & 1 & \rho^1 & \rho^2 \
    \rho^2 & \rho^1 & 1 & \rho^1 \
    \rho^3 & \rho^2 & \rho^1 & 1\
    \end{bmatrix} }
```

Similarly, the arrow-and-lag notation can be used to specify a SAR representing a conventional structural equation model (SEM), cross-lagged (a.k.a. vector autoregressive) models (VAR), dy-namic factor analysis (DFA), or many other time-series models.

#### Value

A reticular action module (RAM) describing dependencies

## Examples

```
# Univariate AR1
dsem = "
    X -> X, 1, rho
    X <-> X, 0, sigma
```

make\_dsem\_ram

```
,,
make_dsem_ram( dsem=dsem, variables="X", times=1:4 )
# Univariate AR2
dsem = "
  X -> X, 1, rho1
 X -> X, 2, rho2
 X <-> X, 0, sigma
make_dsem_ram( dsem=dsem, variables="X", times=1:4 )
# Bivariate VAR
dsem = "
  X -> X, 1, XtoX
 X -> Y, 1, XtoY
  Y -> X, 1, YtoX
  Y -> Y, 1, YtoY
  X <-> X, 0, sdX
 Y <-> Y, 0, sdY
make_dsem_ram( dsem=dsem, variables=c("X","Y"), times=1:4 )
# Dynamic factor analysis with one factor and two manifest variables
# (specifies a random-walk for the factor, and miniscule residual SD)
dsem = "
  factor -> X, 0, loadings1
  factor -> Y, 0, loadings2
  factor -> factor, 1, NA, 1
  X <-> X, 0, NA, 0
                             # No additional variance
  Y <-> Y, Ø, NA, Ø
                             # No additional variance
,,
make_dsem_ram( dsem=dsem, variables=c("X","Y","factor"), times=1:4 )
# ARIMA(1,1,0)
dsem = "
  factor -> factor, 1, rho1 # AR1 component
 X -> X, 1, NA, 1
                            # Integrated component
 factor -> X, 0, NA, 1
                            # No additional variance
 X <-> X, Ø, NA, Ø
,,
make_dsem_ram( dsem=dsem, variables=c("X","factor"), times=1:4 )
# ARIMA(0,0,1)
dsem = "
  factor -> X, 0, NA, 1
  factor -> X, 1, rho1
                        # MA1 component
 X <-> X, 0, NA, 0
                         # No additional variance
make_dsem_ram( dsem=dsem, variables=c("X","factor"), times=1:4 )
```

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make\_eof\_ram

## Description

make\_eof\_ram converts SEM arrow notation to ram describing SEM parameters

# Usage

```
make_eof_ram(
   times,
   variables,
   n_eof,
   remove_na = TRUE,
   standard_deviations = "unequal"
)
```

## Arguments

times	A character vector listing the set of times in order	
variables	A character vector listing the set of variables	
n_eof	Number of EOF modes of variability to estimate	
remove_na	Boolean indicating whether to remove NA values from RAM (default) or not. remove_NA=FALSE might be useful for exploration and diagnostics for advanced users	
standard_deviations		
	One of "equal", "unequal", or a numeric vector indicating fixed values.	

#### Value

A reticular action module (RAM) describing dependencies

## Examples

```
# Two EOFs for two variables
make_eof_ram( times = 2010:2020, variables = c("pollock","cod"), n_eof=2 )
```

make\_sem\_ram

Make a RAM (Reticular Action Model) from a SEM (structural equation model)

# Description

make\_sem\_ram converts SEM arrow notation to ram describing SEM parameters

## Usage

```
make_sem_ram(sem, variables, quiet = FALSE, covs = variables)
```

## Arguments

sem	structural equation model structure, passed to either specifyModel or specifyEquations and then parsed to control the set of path coefficients and variance-covariance parameters
variables	A character vector listing the set of variables
quiet	if FALSE, the default, then the number of input lines is reported and a message is printed suggesting that specifyEquations or cfa be used.
COVS	optional: a character vector of one or more elements, with each element giv- ing a string of variable names, separated by commas. Variances and covariances among all variables in each such string are added to the model. For confirmatory factor analysis models specified via cfa, covs defaults to all of the factors in the model, thus specifying all variances and covariances among these factors. <i>Warn- ing</i> : covs="x1, x2" and covs=c("x1", "x2") are <i>not</i> equivalent: covs="x1, x2" specifies the variance of x1, the variance of x2, <i>and</i> their covariance, while covs=c("x1", "x2") specifies the variance of x1 and the variance of x2 <i>but not</i> their covariance.

# Value

An S3-class "sem\_ram" containing:

model Output from specifyEquations or specifyModel that defines paths and parameters

ram reticular action module (RAM) describing dependencies

parse\_path

## Description

parse\_path is copied from sem::parse.path

#### Usage

```
parse_path(path)
```

## Arguments

path

character string indicating a one-headed or two-headed path in a structural equation model

#### Details

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#### Value

Tagged-list defining variables and direction for a specified path coefficient

predict.tinyVAST Predict using vector autoregressive spatio-temporal model

## Description

Predicts values given new covariates using a tinyVAST model

## Usage

```
## S3 method for class 'tinyVAST'
predict(
   object,
   newdata,
   remove_origdata = FALSE,
   what = c("mu_g", "p_g", "palpha_g", "pgamma_g", "pepsilon_g", "pomega_g", "pdelta_g",
      "pxi_g", "p2_g", "palpha2_g", "pgamma2_g", "pepsilon2_g", "pomega2_g", "pdelta2_g",
      "pxi2_g"),
   se.fit = FALSE,
   ...
)
```

# print.tinyVAST

## Arguments

object	Output from tinyVAST().
newdata	New data-frame of independent variables used to predict the response.
remove_origdat	a
	Whether to eliminate the original data from the TMB object, thereby speed- ing up the TMB object construction. However, this also eliminates information about random-effect variance, and is not appropriate when requesting predictive standard errors or epsilon bias-correction.
what	What REPORTed object to output, where mu_g is the inverse-linked transformed predictor including both linear components, p_g is the first linear predictor, palpha_g is the first predictor from fixed covariates in formula, pgamma_g is the first predictor from random covariates in formula (e.g., splines), pomega_g is the first predictor from spatial variation, pepsilon_g is the first predictor from spatial variation, pepsilon_g is the first predictor from spatial variation, pepsilon_g is the second predictor from fixed covariates in formula, pgamma2_g is the second predictor from fixed covariates in formula, pgamma2_g is the second predictor from spatial variation, pepsilon2_g is the second predictor from spatial variation, pepsilon2_g is the second predictor from spatial variation, and pxi2_g is the second predictor from spatially varying coefficients.
se.fit	Calculate standard errors?
	Not used.

#### Value

Either a vector with the prediction for each row of newdata, or a named list with the prediction and standard error (when se.fit = TRUE).

print.tinyVAST print summary of tinyVAST model

## Description

print summary of tinyVAST model

## Usage

## S3 method for class 'tinyVAST'
print(x, ...)

## Arguments

х	output from tinyVAST
	not used

## Value

invisibly returns a named list of key model outputs and summary statements

red\_snapper Presence/absence, count, and biomass data for red snapper

## Description

Data used to demonstrate and test analysis using multiple data types

## Usage

data(red\_snapper)

red\_snapper\_shapefile Shapefile for red snapper analysis

#### Description

Spatial extent used for red snapper analysis, derived from Chap-7 of doi:10.1201/9781003410294

## Usage

```
data(red_snapper_shapefile)
```

reload\_model Reload a previously fitted model

#### Description

reload\_model allows a user to save a fitted model, reload it in a new R terminal, and then relink the DLLs so that it functions as expected.

## Usage

```
reload_model(x, check_gradient = TRUE)
```

## Arguments

x Output from tinyVAST, potentially with DLLs not linked check\_gradient Whether to check the gradients of the reloaded model

## Value

Output from tinyVAST with DLLs relinked

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residuals.tinyVAST Calculate deviance or response residuals for tinyVAST

#### Description

Calculate residuals

## Usage

```
## S3 method for class 'tinyVAST'
residuals(object, type = c("deviance", "response"), ...)
```

## Arguments

object	Output from tinyVAST()
type	which type of residuals to compute (only option is "deviance" or "response" for now)
	Note used

## Value

a vector residuals, associated with each row of data supplied during fitting

rmvnorm_prec Multivariate Normal Random Deviates using Sparse Precision	
---	--

#### Description

This function provides a random number generator for the multivariate normal distribution with mean equal to mean and sparse precision matrix Q.

## Usage

rmvnorm\_prec(Q, n = 1, mean = rep(0, nrow(Q)))

## Arguments

Q	sparse precision (inverse-covariance) matrix.
n	number of observations.
mean	mean vector.

#### Value

a matrix with dimension length(mean) by n, containing realized draws from the specified mean and precision

rotate\_pca

## Description

Rotate lower-triangle loadings matrix to order factors from largest to smallest variance.

## Usage

```
rotate_pca(
  L_tf,
  x_sf = matrix(0, nrow = 0, ncol = ncol(L_tf)),
  order = c("none", "increasing", "decreasing")
)
```

## Arguments

L_tf	Loadings matrix with dimension $T \times F$ .
x_sf	Spatial response with dimensions $S \times F$ .
order	Options for resolving label-switching via reflecting each factor to achieve a given order across dimension $T$ .

#### Value

List containing the rotated loadings  $L_tf$ , the inverse-rotated response matrix  $x_sf$ , and the rotation H

salmon\_returns North Pacific salmon returns

#### Description

Data used to demonstrate and test multivariate second-order autoregressive models using a simultaneous autoregressive (SAR) process across regions. Data are from doi:10.1002/mcf2.10023

#### Usage

data(salmon\_returns)

#### Description

sample\_variable samples from the joint distribution of random and fixed effects to approximate the predictive distribution for a variable

Using sample\_fixed=TRUE (the default) in sample\_variable propagates variance in both fixed and random effects, while using sample\_fixed=FALSE does not. Sampling fixed effects will sometimes cause numerical under- or overflow (i.e., output values of NA) in cases when variance parameters are estimated imprecisely. In these cases, the multivariate normal approximation being used is a poor representation of the tail probabilities, and results in some samples with implausibly high (or negative) variances, such that the associated random effects then have implausibly high magnitude.

## Usage

```
sample_variable(
    x,
    variable_name = "mu_i",
    n_samples = 100,
    sample_fixed = TRUE,
    seed = 123456
)
```

#### Arguments

х	output from \code{tinyVAST()}
variable_name	<pre>name of variable available in report using Obj\$report() or parameters using Obj\$env\$parList()</pre>
n_samples	number of samples from the joint predictive distribution for fixed and random effects. Default is 100, which is slow.
sample_fixed	whether to sample fixed and random effects, sample_fixed=TRUE as by default, or just sample random effects, sample_fixed=FALSE
seed	integer used to set random-number seed when sampling variables, as passed to set.seed(.)

## Value

A matrix with a row for each data supplied during fitting, and n\_samples columns, where each column in a vector of samples for a requested quantity given sampled uncertainty in fixed and/or random effects

## Examples

sea\_ice

Arctic September sea ice concentrations

## Description

Data used to demonstrate and test empirical orthogonal function generalized linear latent variable model (EOF-GLLVM)

#### Usage

data(sea\_ice)

sfnetwork\_evaluator Construct projection matrix for stream network

## Description

Make sparse matrix to project from stream-network nodes to user-supplied points

#### Usage

```
sfnetwork_evaluator(stream, loc, tolerance = 0.01)
```

#### Arguments

stream	sfnetworks object representing stream network
loc	$\mathbf{sf}$ object representing points to which are being projected
tolerance	error-check tolerance

## Value

the sparse interpolation matrix, with rows for each row of data supplied during fitting and columns for each spatial random effect.

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sfnetwork\_mesh

#### Description

make an object representing spatial information required to specify a stream-network spatial domain, similar in usage to link[fmesher]{fm\_mesh\_2d} for a 2-dimensional continuous domain

#### Usage

```
sfnetwork_mesh(stream)
```

## Arguments

stream sfnetworks object representing stream network

## Value

An object (list) of class sfnetwork\_mesh. Elements include:

N The number of random effects used to represent the network

**table** a table containing a description of parent nodes (from), childen nodes (to), and the distance separating them

stream copy of the stream network object passed as argument

simulate.tinyVAST Simulate new data from a fitted model

#### Description

simulate.tinyVAST is an S3 method for producing a matrix of simulations from a fitted model. It can be used with the **DHARMa** package among other uses. Code is modified from the version in sdmTMB

#### Usage

```
## S3 method for class 'tinyVAST'
simulate(
   object,
   nsim = 1L,
   seed = sample.int(1e+06, 1L),
   type = c("mle-eb", "mle-mvn"),
   ...
)
```

#### Arguments

object	output from tinyVAST()
nsim	how many simulations to do
seed	random seed
type	How parameters should be treated. "mle-eb": fixed effects are at their max- imum likelihood (MLE) estimates and random effects are at their empirical Bayes (EB) estimates. "mle-mvn": fixed effects are at their MLEs but ran- dom effects are taken from a single approximate sample. This latter option is a suggested approach if these simulations will be used for goodness of fit testing (e.g., with the DHARMa package).
•••	not used

#### Value

A matrix with row for each row of data in the fitted model and nsim columns, containing new samples from the fitted model.

## Examples

simulate\_sfnetwork Simulate GMRF for stream network

## Description

Simulate values from a GMRF using a tail-up exponential model on a stream network

## Usage

```
simulate_sfnetwork(sfnetwork_mesh, theta, n = 1, what = c("samples", "Q"))
```

## summary.tinyVAST

## Arguments

sfnetwork_mesh	Output from sfnetwork_mesh
theta	Decorrelation rate
n	number of simulated GMRFs
what	Whether to return the simulated GMRF or its precision matrix

#### Value

a matrix of simulated values for a Gaussian Markov random field arising from a stream-network spatial domain, with row for each spatial random effect and n columns, using the sparse precision matrix defined in Charsley et al. (2023)

## References

Charsley, A. R., Gruss, A., Thorson, J. T., Rudd, M. B., Crow, S. K., David, B., Williams, E. K., & Hoyle, S. D. (2023). Catchment-scale stream network spatio-temporal models, applied to the freshwater stages of a diadromous fish species, longfin eel (Anguilla dieffenbachii). Fisheries Research, 259, 106583. doi:10.1016/j.fishres.2022.106583

summary.tinyVAST summarize tinyVAST

## Description

summarize parameters from a fitted tinyVAST

## Usage

```
## S3 method for class 'tinyVAST'
summary(
   object,
   what = c("space_term", "time_term", "spacetime_term", "fixed"),
   predictor = c("one", "two"),
   ...
)
```

#### Arguments

object	Output from tinyVAST()
what	What component to summarize, whether space_term, spacetime_term, or fixed for the fixed effects included in the GAM formula
predictor	whether to get the 1st or 2nd linear predictor (the latter is only applicable in delta models)
	Not used

#### Details

tinyVAST includes three components:

- **Space-variable interaction** a separable Gaussian Markov random field (GMRF) constructed from a structural equation model (SEM) and a spatial variable
- **Space-variable-time interaction** a separable GMRF constructed from a a dynamic SEM (a nonseparable time-variable interaction) and a spatial variable

Additive variation a generalized additive model (GAM), representing exogenous covariates

Each of these are summarized and interpreted differently, and summary.tinyVAST facilitates this.

Regarding the DSEM componennt, tinyVAST includes an "arrow and lag" notation, which specifies the set of path coefficients and exogenous variance parameters to be estimated. Function tinyVAST then estimates the maximum likelihood value for those coefficients and parameters by maximizing the log-marginal likelihood.

However, many users will want to associate individual parameters and standard errors with the path coefficients that were specified using the "arrow and lag" notation. This task is complicated in models where some path coefficients or variance parameters are specified to share a single value a priori, or were assigned a name of NA and hence assumed to have a fixed value a priori (such that these coefficients or parameters have an assigned value but no standard error). The summary function therefore compiles the MLE for coefficients (including duplicating values for any path coefficients that assigned the same value) and standard error estimates, and outputs those in a table that associates them with the user-supplied path and parameter names. It also outputs the z-score and a p-value arising from a two-sided Wald test (i.e. comparing the estimate divided by standard error against a standard normal distribution).

#### Value

A data-frame containing the estimate (and standard errors, two-sided Wald-test z-value, and associated p-value if the standard errors are available) for model parameters, including the fixed-effects specified via formula, or the path coefficients for the spatial SEM specified via space\_term, the dynamic SEM specified via time\_term, or the spatial dynamic SEM specified via spacetime\_term

tinyVAST

Fit vector autoregressive spatio-temporal model

#### Description

Fits a vector autoregressive spatio-temporal (VAST) model using a minimal feature-set and a widely used interface.

#### Usage

```
tinyVAST(
  formula,
  data,
  time_term = NULL,
```

# tinyVAST

```
space_term = NULL,
spacetime_term = NULL,
family = gaussian(),
space_columns = c("x", "y"),
spatial_domain = NULL,
time_column = "time",
times = NULL,
variable_column = "var",
variables = NULL,
distribution_column = "dist",
delta_options = list(formula = ~1),
spatial_varying = NULL,
weights = NULL,
control = tinyVASTcontrol(),
...
```

# Arguments

)

formula	Formula with response on left-hand-side and predictors on right-hand-side, parsed by $mgcv$ and hence allowing $s(.)$ for splines or $offset(.)$ for an offset.
data	Data-frame of predictor, response, and offset variables. Also includes variables that specify space, time, variables, and the distribution for samples, as identified by arguments variable_column, time_column, space_columns, and distribution_column.
time_term	Specification for time-series structural equation model structure for construct- ing a time-variable interaction that defines a time-varying intercept for each variable (i.e., applies uniformly across space). time_term=NULL disables the space-variable interaction; see make_dsem_ram() for notation.
space_term	Specification for structural equation model structure for constructing a space- variable interaction. space_term=NULL disables the space-variable interaction; see make_sem_ram() for notation.
<pre>spacetime_term</pre>	Specification for time-series structural equation model structure including lagged or simultaneous effects for constructing a time-variable interaction, which is then combined in a separable process with the spatial correlation to form a space-time-variable interaction (i.e., the interaction occurs locally at each site). spacetime_term=NULL disables the space-variable interaction; see make_dsem_ram() or make_eof_ram().
family	A function returning a class family, including gaussian(), lognormal(), tweedie(), binomial(), Gamma(), poisson(), nbinom1(), or nbinom2(). Alternatively, can be a named list of these functions, with names that match levels of data\$distribution_column to allow different families by row of data. Delta model families are possible, and see Families for delta-model options,
space_columns	A string or character vector that indicates the column(s) of data indicating the location of each sample. When spatial_domain is an igraph object, space_columns is a string with with levels matching the names of vertices of that object. When spatial_domain is an fmesher or sfnetwork object, space_columns is a character vector indicating columns of data with coordinates for each sample.

spatial_domain	Object that represents spatial relationships, either using fmesher::fm_mesh_2d() to apply the SPDE method, igraph::make_empty_graph() for independent time-series, igraph::make_graph() to apply a simultaneous autoregressive (SAR) process, sfnetwork_mesh() for stream networks, or NULL to specify a single site. If using igraph then the graph must have vertex names V(graph)\$name that match levels of data[,'space_columns']	
time_column	A character string indicating the column of data listing the time-interval for each sample, from the set of times in argument times.	
times	A integer vector listing the set of times in order. If times=NULL, then it is filled in as the vector of integers from the minimum to maximum value of data\$time.	
variable_column		
	A character string indicating the column of data listing the variable for each sample, from the set of times in argument variables.	
variables	A character vector listing the set of variables. if variables=NULL, then it is filled in as the unique values from data\$variable_columns.	
distribution_co	plumn	
	A character string indicating the column of data listing the distribution for each sample, from the set of names in argument family. if variables=NULL, then it is filled in as the unique values from data\$variables.	
delta_options	a named list with slots for formula, space_term, and spacetime_term. These specify options for the second linear predictor of a delta model, and are only used (or estimable) when a delta family is used for some samples.	
spatial_varying		
	a formula specifying spatially varying coefficients.	
weights	A numeric vector representing optional likelihood weights for the data likelihood. Weights do not have to sum to one and are not internally modified. Thee weights argument needs to be a vector and not a name of the variable in the data frame.	
control	Output from tinyVASTcontrol(), used to define user settings.	
	Not used.	

## Details

tinyVAST includes four basic inputs that specify the model structure:

- formula specifies covariates and splines in a Generalized Additive Model;
- space\_term specifies interactions among variables and over time, constructing the space-variable interaction.
- spacetime\_term specifies interactions among variables and over time, constructing the space-time-variable interaction.
- spatial\_domain specifies spatial correlations

the default spacetime\_term=NULL and space\_term=NULL turns off all multivariate and temporal indexing, such that spatial\_domain is then ignored, and the model collapses to a generalized additive model using gam. To specify a univariate spatial model, the user must specify spatial\_domain

#### tinyVAST

and either space\_term="" or spacetime\_term="", where the latter two are then parsed to include a single exogenous variance for the single variable

#### Model type

Generalized additive model

Dynamic structural equation model (including vector autoregressive, dynamic factor analysis, ARIMA, and structural equation Univariate spatio-temporal model, or multiple independence spatio-temporal variables Multivariate spatial model including interactions

Vector autoregressive spatio-temporal model (i.e., lag-1 interactions among variables)

#### Value

An object (list) of class tinyVAST. Elements include:

data Data-frame supplied during model fitting

spatial\_domain the spatial domain supplied during fitting

formula the formula specified during model fitting

obj The TMB object from MakeADFun

**opt** The output from nlminb

opt The report from obj\$report()

sdrep The output from sdreport

tmb\_inputs The list of inputs passed to MakeADFun

call A record of the function call

run\_time Total time to run model

interal Objects useful for package function, i.e., all arguments passed during the call

deviance\_explained output from deviance\_explained

#### See Also

Details section of make\_dsem\_ram() for a summary of the math involved with constructing the DSEM, and doi:10.1111/2041210X.14289 for more background on math and inference

doi:10.48550/arXiv.2401.10193 for more details on how GAM, SEM, and DSEM components are combined from a statistical and software-user perspective

summary.tinyVAST() to visualize parameter estimates related to SEM and DSEM model components

#### Examples

```
# Simulate a seperable two-dimensional AR1 spatial process
n_x = n_y = 25
n_w = 10
R_xx = exp(-0.4 * abs(outer(1:n_x, 1:n_x, FUN="-")))
R_yy = exp(-0.4 * abs(outer(1:n_y, 1:n_y, FUN="-")))
z = mvtnorm::rmvnorm(1, sigma=kronecker(R_xx,R_yy))
```

tinyVASTcontrol

Control parameters for tinyVAST

#### Description

Control parameters for tinyVAST

## Usage

```
tinyVASTcontrol(
  nlminb_loops = 1,
  newton_loops = 0,
  eval.max = 1000,
  iter.max = 1000,
  getsd = TRUE,
  silent = getOption("tinyVAST.silent", TRUE),
  trace = getOption("tinyVAST.trace", 0),
  verbose = getOption("tinyVAST.verbose", FALSE),
  profile = c(),
  tmb_par = NULL,
  tmb_map = NULL,
  gmrf_parameterization = c("separable", "projection"),
  reml = FALSE,
  getJointPrecision = FALSE,
 calculate_deviance_explained = TRUE,
  run_model = TRUE
)
```

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# tinyVASTcontrol

# Arguments

nlminb_loops	Integer number of times to call stats::nlminb().	
newton_loops	Integer number of Newton steps to do after running stats::nlminb().	
eval.max	Maximum number of evaluations of the objective function allowed. Passed to control in stats::nlminb().	
iter.max	Maximum number of iterations allowed. Passed to control in stats::nlminb().	
getsd	Boolean indicating whether to call TMB::sdreport()	
silent	Disable terminal output for inner optimizer?	
trace	Parameter values are printed every trace iteration for the outer optimizer. Passed to control in stats::nlminb().	
verbose	Output additional messages about model steps during fitting?	
profile	Parameters to profile out of the likelihood (this subset will be appended to random with Laplace approximation disabled).	
tmb_par	list of parameters for starting values, with shape identical to tiny VAST() $internal$	
tmb_map	input passed to TMB::MakeADFun as argument map, over-writing the version tinyVAST()\$tmb_inputs\$tmb_map and allowing detailed control over estimated parameters (advanced feature)	
gmrf_parameter:	ization	
	Parameterization to use for the Gaussian Markov random field, where the default separable constructs a full-rank and separable precision matrix, and the alter- native projection constructs a full-rank and IID precision for variables over time, and then projects this using the inverse-cholesky of the precision, where this projection allows for rank-deficient covariance.	
reml	Logical: use REML (restricted maximum likelihood) estimation rather than maximum likelihood? Internally, this adds the fixed effects to the list of random effects to integrate over.	
getJointPrecision		
	whether to get the joint precision matrix. Passed to sdreport.	
calculate_deviance_explained whether to calculate proportion of deviance explained. See deviance_explained()		
run_model	whether to run the model of export TMB objects prior to compilation (useful for debugging)	

# Value

An object (list) of class tinyVASTcontrol, containing either default or updated values supplied by the user for model settings

vcov.tinyVAST

## Description

extract the covariance of fixed effects, or both fixed and random effects.

# Usage

```
## S3 method for class 'tinyVAST'
vcov(object, which = c("fixed", "random", "both"), ...)
```

## Arguments

object	output from tinyVAST()
which	whether to extract the covariance among fixed effects, random effects, or both
	ignored, for method compatibility

#### Value

A square matrix containing the estimated covariances among the parameter estimates in the model. The dimensions dependend upon the argument which, to determine whether fixed, random effects, or both are outputted.

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